M3 - June 2007

1- a) $A = \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx = \left[x^2 - \frac{1}{3}x^3\right]_0^2 = 4 - \frac{8}{3}$

$= \frac{4}{3}$ units^2

b) By symmetry $\bar{x} = 1$

$\frac{4}{3}\bar{y} = \int_0^2 \frac{1}{2}y^2\,dx = \frac{1}{2}\int_0^2 (2x - x^2)^2\,dx = \int_0^2 \frac{1}{2}(4x^2 - 4x^3 + x^4)\,dx$

$= \frac{1}{2}\left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5\right]_0^2 \quad \frac{1}{2}\left(\frac{32}{3} - 16 + \frac{32}{5}\right) = \frac{16}{15} \times \frac{1}{2} = \frac{8}{15}$

$\bar{y} = \frac{3 \times 8}{4 \times 5} = \frac{2}{5}$

∴ Centre of mass of lamina is at $\left(1, \frac{2}{5}\right)$

2- a) CoM of cylinder: $\frac{1}{2}h$ from O

$\frac{1}{2} h \times 2\pi h^2 h = (2\pi h^2 h + \pi h^2)\bar{d}$

$\frac{1}{2} h \times 2 = (2+1)\bar{d}$

$h = 3\bar{d}$

$\bar{d} = \frac{1}{3} h$

b) $\frac{1}{3} hm + \frac{1}{2} hm = 2m\bar{d}$

$\frac{5}{6} h = 2\bar{d}$

$\bar{d} = \frac{5}{12} h$

3- a) $F = \frac{k}{x^2}$ When $x = R$, $F = gm$ $gm = \frac{k}{R^2}$ $k = mgR^2$

b) $[F = ma]$

$\frac{-mgR^2}{x^2} = mv\frac{dv}{dx}$

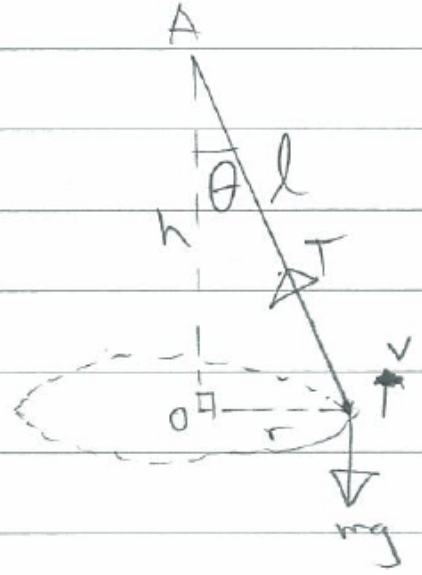
$-gR^2 \int_{2R}^{R} x^{-2} dx = \int_0^v v\,dv$

$+gR^2 \left[\frac{1}{x}\right]_{2R}^{R} = \frac{1}{2}\left[v^2\right]_0^v$

$gR - \frac{1}{2}gR = \frac{1}{2}v^2$

$v^2 = gR$ $v = \sqrt{gR}$ ms^{-1}

4-



$\uparrow mg = T\cos\theta$ $\leftarrow [F = ma]$

T ↳ $\frac{mg}{\cos\theta}$ (i) $\quad T\sin\theta = \frac{mv^2}{r}$ (ii)

(i) ÷ (ii): $\frac{mg}{\cos\theta}\cdot\sin\theta = \frac{mv^2}{r}$

$g\tan\theta = \frac{v^2}{r}$ (iii)

$\tan\theta = \frac{r}{h} = \frac{r}{\sqrt{l^2-r^2}}$ (iv) $\quad$ (iv) × (ii): $\frac{gr}{\sqrt{l^2-r^2}} = \frac{v^2}{r}$

$gr^2 = v^2\sqrt{l^2-r^2}$

5- a) $\ddot{x} = -w^2x$

$|-1| = |-w^2\cdot 0.04|$

$w^2 = 25$

$w = 5$

$T = \frac{2\pi}{w} = \frac{2\pi}{5}$

c) $x = a\sin st$

b) $v^2 = \omega^2(a^2 - x^2)$

$0.2^2 = 25(a^2 - 0.04^2)$

$0.04 = 25a^2 - 0.04$

$a^2 = \frac{0.08}{25}$

$a = \frac{\sqrt{2}}{25}\ m$

$\frac{1}{2}a = a\sin 5t$

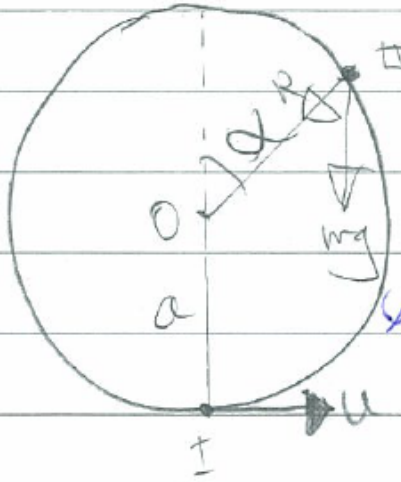
$\pm\frac{1}{2} = \sin 5t$

$5t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$t = \frac{\pi}{30}, \frac{5\pi}{30}, \frac{7\pi}{30}, \frac{11\pi}{30}$

$T_{me} = \left(\frac{5\pi}{30} - \frac{\pi}{30}\right) + \left(\frac{11\pi}{30} - \frac{7\pi}{30}\right) = \frac{4\pi}{15}$

6-



a) $mE_A = mE_B$

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(a + a\cos\alpha)$

$u^2 = v^2 + 2ga + 2ga\cos\alpha$ ①

$[F = ma]$ $R + mg\cos\alpha = \frac{mv^2}{r}$

$mg\cos\alpha = \frac{mv^2}{a}$

$v^2 = ag\cos\alpha$ ②

(2) in (1): $U^2 = ag\cos\alpha + 2ga + 2ga\cos\alpha$

$= 3ag\cos\alpha + 2ag = ag(2 + 3\cos\alpha)$

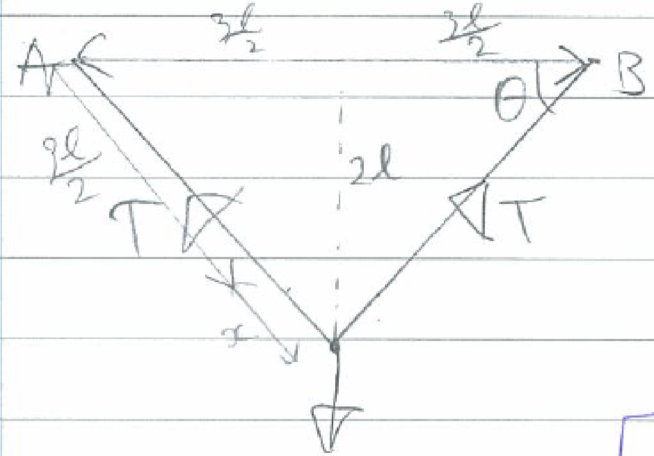
b) $ME_T = ME_0$

$\frac{1}{2}mU^2 = \frac{1}{2}mW^2 + mga$

$W^2 = U^2 - 2ga = 2ga + 3ga\cos\alpha - 2ga$

$= 3ga \times \frac{1}{\sqrt{3}} = \frac{3ag}{\sqrt{3}} = ag\sqrt{3}$

7_



a) ↑ $mg = 2T\sin\theta$ (1)

$\frac{3l}{2} + x = \sqrt{(2l)^2 + \left(\frac{3l}{2}\right)^2} = \sqrt{4l^2 + \frac{9}{4}l^2}$

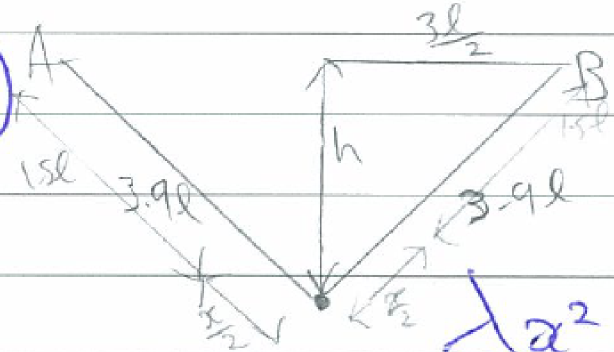
$x = \frac{5}{2}l - \frac{3l}{2} = l$

mg (HL) $T = \frac{\lambda x}{a} = \frac{\lambda x \cdot 2}{3l} = \frac{2\lambda}{3}$

② in ①: $mg = 2 \times \frac{2\lambda}{3} \sin\theta$ $\sin\theta = \frac{2l}{\frac{5l}{2}} = \frac{4}{5}$ ④

$\frac{3mg}{4\sin\theta} = \lambda$ ③   ④ in ③: $\lambda = \frac{3mg}{4 \times \frac{4}{5}} = \frac{15mg}{16}$

b)



$h^2 = (3.9^2 - 1.5^2)l^2 = 12.96l^2$

$h = 3.6l$

$x = (7.8 - 3)l$

$= 4.8l$

$\frac{\lambda x^2}{2a} = mgh + \frac{1}{2}mv^2$

$\frac{15mg}{16} \cdot \frac{(4.8l)^2}{6l} = mg \times 3.6l + \frac{1}{2}mv^2$

$v^2 = \frac{15 \times 4.8^2 \times 2lg}{16 \times 6} - 2 \times 3.6lg = 70.56l - 70.56l$

$= 0$